

## Problem Set #1

Due in lecture Wednesday, July 5<sup>th</sup>, 2017. *No late problem sets accepted for credit! (Seriously.)*

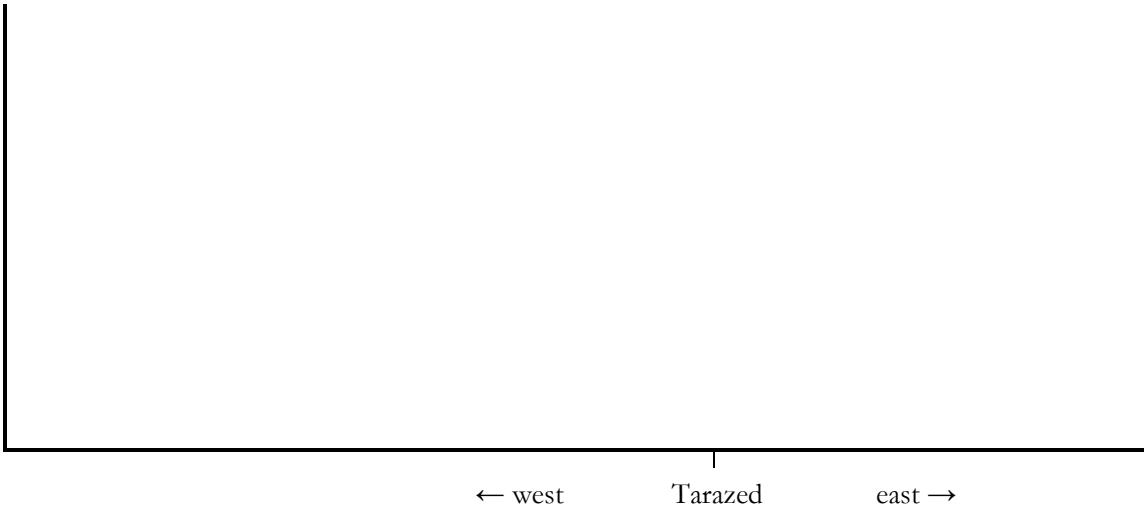
NOTE: To ensure proper grading, write your answers in the area indicated.

1. **End of Line** (7 points) The city of Tarazed lies at the eastern end of a rail line. To the west are two hundred miles of wide plains along which there are several towns—all connected to Tarazed by the rail line. To the east of Tarazed are one hundred miles of mountains, whose small villages are connected by a few winding dirt roads.

The only factory in the region is in Tarazed. It produces widgets at a cost of \$40 per unit at the factory door. Thanks to the rail line, the transport cost to anywhere west of Tarazed is \$0.50 per mile. Given the poor state of the roads through the mountains, the transport cost to anywhere east of Tarazed is \$2.00 per mile.

a. Graph the costs of widgets produced throughout the entire 300-mile region. Carefully label the costs at the factory, at the far western end of the region, and at the far eastern end of the region.

Cost (in \$)



b. Assume that the cost of making a widget at home is \$100. What is the market area for the factory? (That is, how many miles west of Tarazed will it stretch, and how many miles east of Tarazed will it stretch?) Show your work.

number of miles to the **west** of Tarazed  
in the market area of the factory

number of miles to the **east** of Tarazed  
in the market area of the factory

In reality, one of the key inputs for widgets is easier for the residents of the western plains to acquire. **For all remaining parts of the problem**, assume that the cost of making widgets at home anywhere west of Tarazed is actually \$90, and that the cost of making widgets at home is \$120 anywhere east of Tarazed.

c. Now what is the market area for the factory? (That is, how many miles west of Tarazed will it stretch, and how many miles east of Tarazed will it stretch?) Show your work.

number of miles to the **west** of Tarazed  
in the market area of the factory

number of miles to the **east** of Tarazed  
in the market area of the factory

The owners of the factory would like its market area to extend more broadly, and they are considering two investment options:

- First, they could help the government of the region finance the upgrading and paving of the mountain roads, which would reduce the transport costs east of Tarazed to only \$1.00 per mile. (The transportation costs west of Tarazed would stay at \$0.50 per mile.)
- Second, they could purchase and install expensive new equipment in the factory that would reduce the cost of a widget at the factory door to only \$28 per unit.

Each of these options would cost exactly the same, and the owners of the factory can choose only one of these options. Assume that potential customers for widgets are distributed evenly across the entire 300-mile region.

d. How many additional miles (on either side of Tarazed) would be added to the factory's market area—relative to part (c)—under the first investment option of upgrading and paving the mountain roads? Show your work.

miles added to the factory's market  
area beyond the answers in part (c)

e. How many additional miles (on either side of Tarazed) would be added to the factory's market area—relative to part (c)—under the second investment option of installing new equipment in the factory? Show your work.

miles added to the factory's market area beyond the answers in part (c)

f. Given the answers to parts d) and e) above, which investment option will the factory owners choose? Briefly explain why they will go with that option.

2. **You're a Lumberjack and You're OK** (2 points) You own a firewood business that involves chopping down trees, cutting the trees up into firewood, and transporting them to market. You can *either* cut the trees up into firewood where they are chopped down and transport firewood to the market *or* you can transport the chopped down trees to the market and cut them into firewood there. The costs of the various operations are as follows:

Chopping down trees	\$120 per ton
Cutting trees into firewood	\$100 per ton
Transporting trees to market	\$90 per ton
Transporting firewood to market	\$60 per ton

a. Assuming that no wood is lost in the either chopping down trees or cutting them into firewood, where should the trees be cut into firewood to minimize costs: where the trees are chopped down, or at the market?

b. Explain your answer to part (a). Discuss in your explanation which of the four pieces of data from the set of above that you needed to make the determination and which of the pieces of data were not important.

3. **Go Big or Go Small?** (14 points) Alpheratz Technologies, software company, is looking to expand and to hire programmers to help them make smartphone apps and is trying to figure out where to locate this new team.

There are two options that they are considering:

- Andromedaville is a small town with a fixed, perfectly inelastic supply of 30 programmers who would be willing to work at any price.
- Pegasus City is a major metropolis with a perfectly elastic supply of programmers. The prevailing (and fixed) wage for programmers is \$24 per hour.

Complicating their decision somewhat, Alpheratz Technologies doesn't know how well its apps will sell. Consequently, it doesn't know precisely what its demand for labor—in this case, programmers—will be. It does know that the demand will be either high or low.

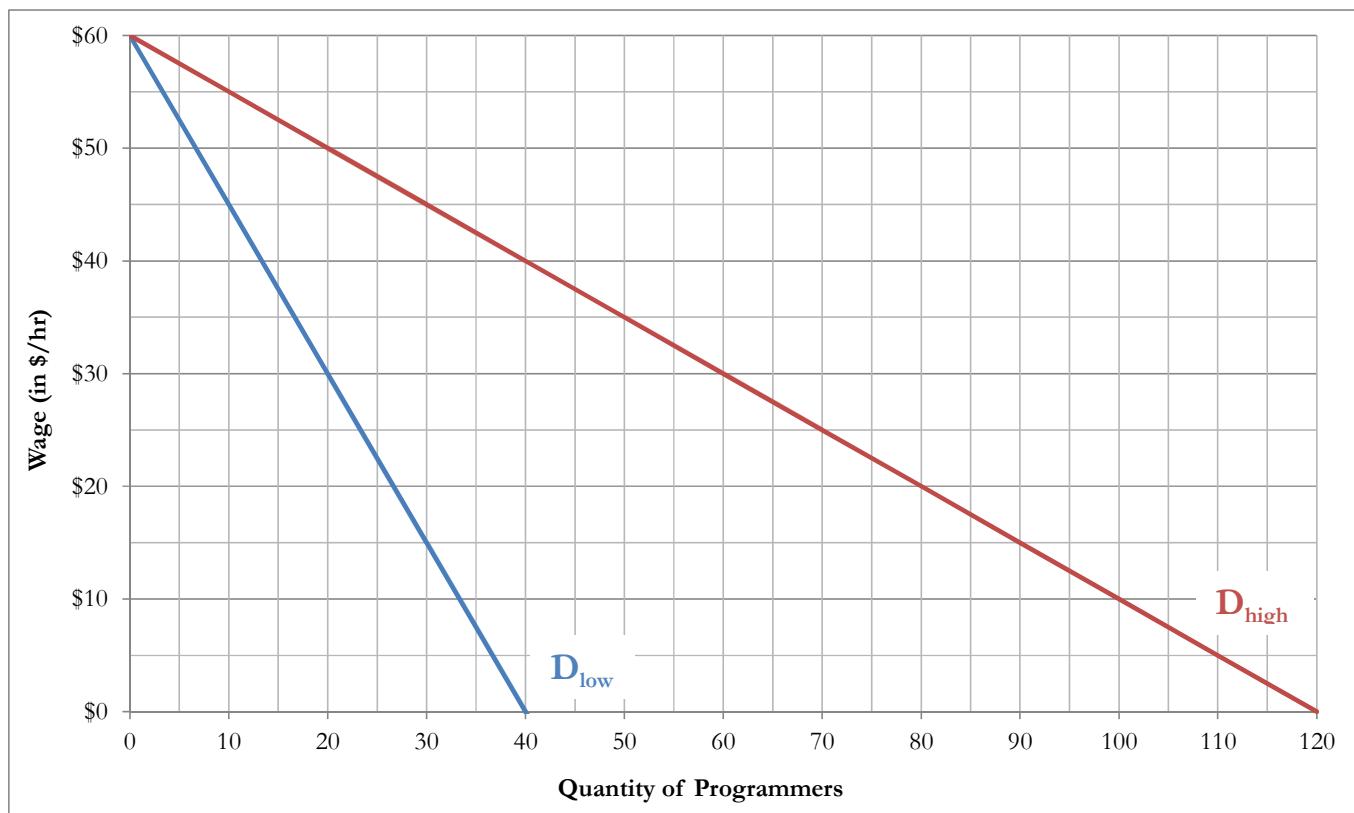
The high-demand curve may be expressed algebraically as:

$$\text{Wage} = \$60 - 0.5 \cdot \text{Quantity}_{\text{High}}$$

Likewise, the low-demand curve may be expressed algebraically as:

$$\text{Wage} = \$60 - 1.5 \cdot \text{Quantity}_{\text{Low}}$$

a. On the graph draw and label the supply curve for labor in Andromedaville.



b. If Alpheratz Technologies were located in Andromedaville and had a **high** demand for labor, how many programmers would it hire? Briefly explain.

number of programmers hired in  
Andromedaville with high demand

c. If Alpheratz Technologies were located in Andromedaville and had a **low** demand for labor, how many programmers would it hire? Briefly explain.

number of programmers hired in  
Andromedaville with low demand

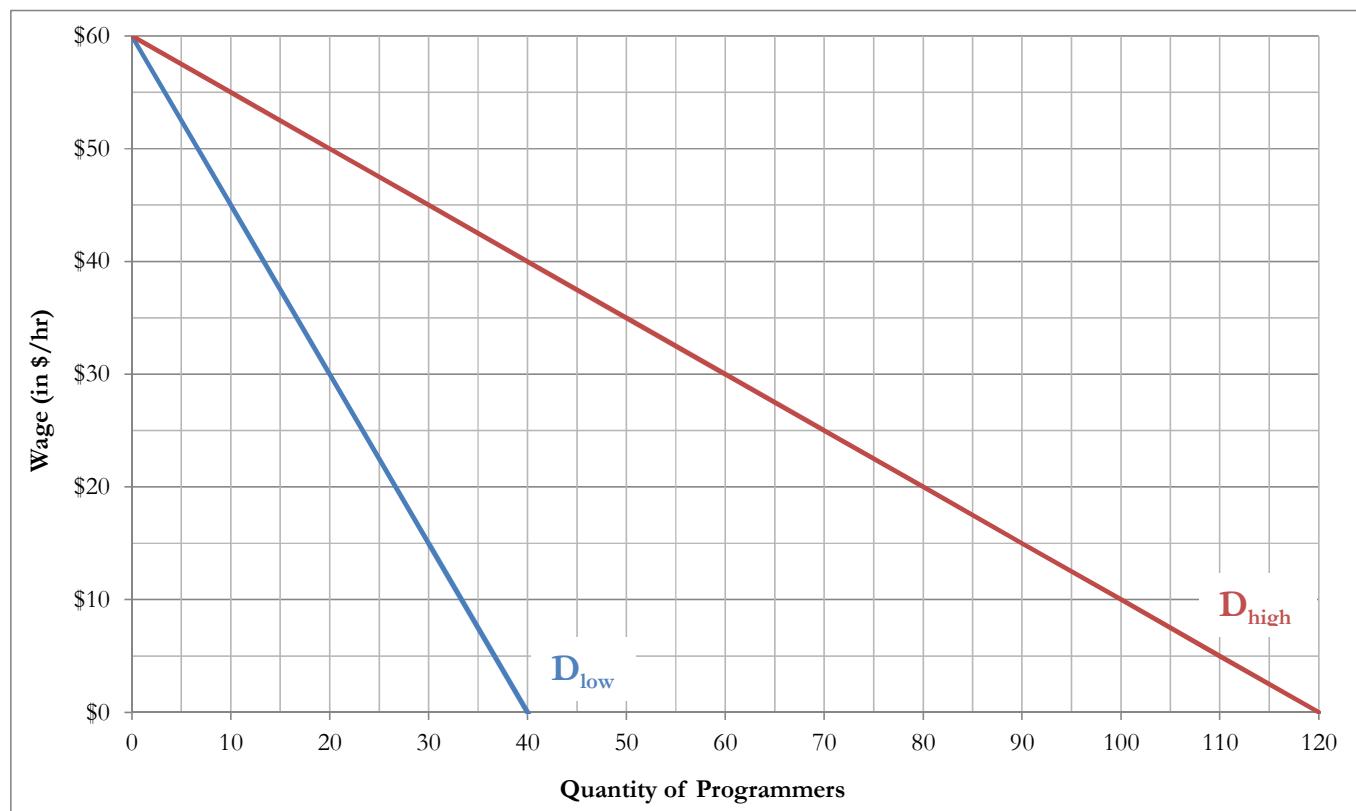
d. How much profit would Alpheratz Technologies earn if it were located in Andromedaville and had a **high** demand for labor? Show your work.

profit in Andromedaville  
with high demand

e. How much profit would Alpheratz Technologies earn if it were located in Andromedaville and had a **low** demand for labor? Show your work.

profit in Andromedaville  
with low demand

f. This time draw and label the supply curve for labor in Pegasus City (given the description above).



g. If Alpheratz Technologies were located in Pegasus City and had a **high** demand for labor, how many programmers would it hire? Show your work.

[Hint: it may be more helpful to look at the algebraic description of the relevant demand curve rather than to try to figure it out from the graph.]

number of programmers hired in  
Pegasus City with high demand

h. If Alpheratz Technologies were located in Pegasus City and had a **low** demand for labor, how many programmers would it hire? Show your work.

[Hint: it may be more helpful to look at the algebraic description of the relevant demand curve rather than to try to figure it out from the graph.]

number of programmers hired in  
Pegasus City with low demand

i. How much profit would Alpheratz Technologies earn if it were located in Pegasus City and had a **high** demand for labor? Show your work.

profit in Pegasus City  
with high demand

j. How much profit would Alpheratz Technologies earn if it were located in Pegasus City and had a **low** demand for labor? Show your work.

profit in Pegasus City  
with low demand

Alpheratz Technologies believes that the probability that it will have high demand for programmers is 10%.

k. Given your answers above, what is the expected profit of Alpheratz Technologies if it were to locate in Andromedaville and the probability of high demand were 10%? Show your work.

expected profit in  
Andromedaville

l. Given your answers above, what is the expected profit of Alpheratz Technologies if it were to locate in Pegasus City and the probability of high demand were 10%? Show your work.

expected profit in  
Pegasus City

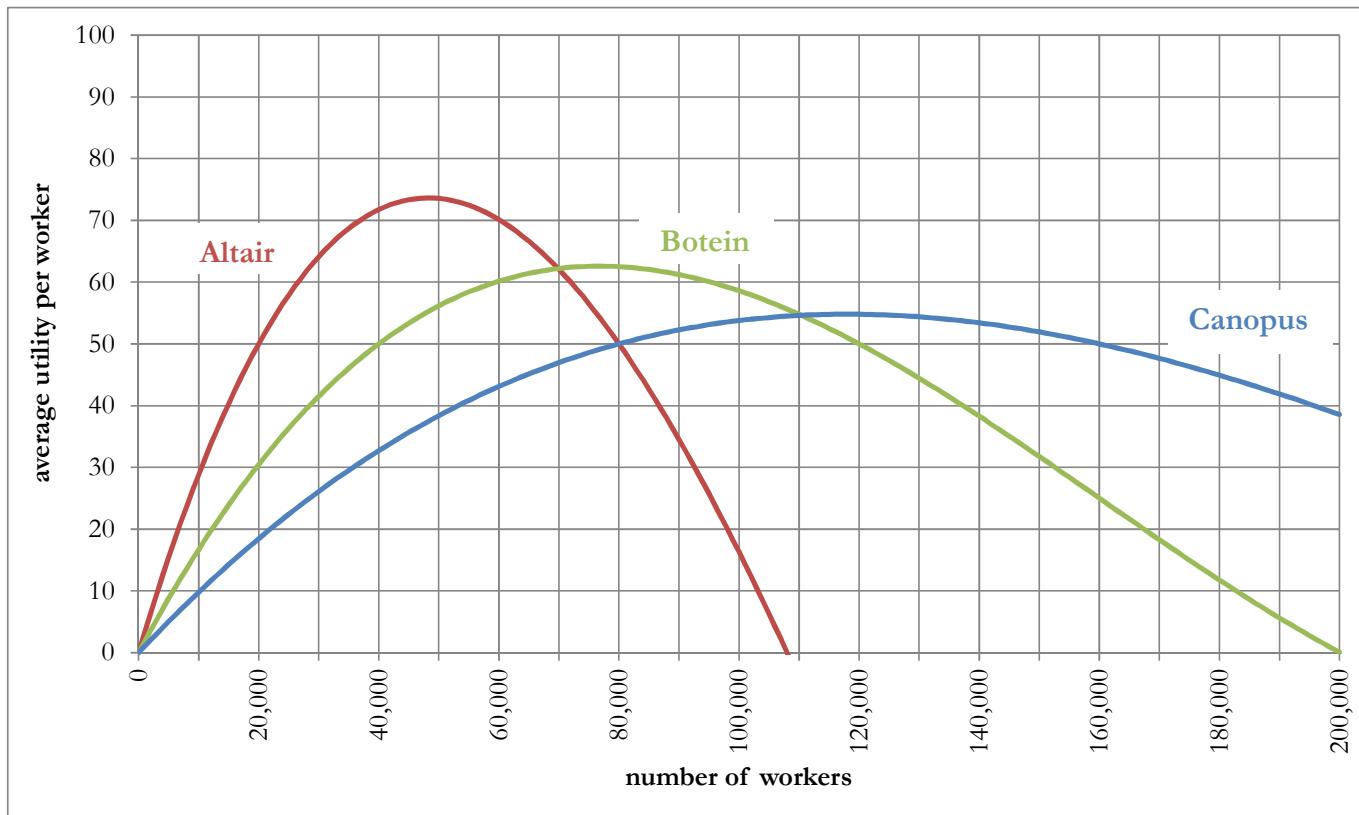
m. Given your last two answers, in which city will Alpheratz Technologies decide to locate its new team? Why?

n. What would the probability of high demand— $p$ —have to be for the expected profit to be the same in both Andromedaville and Pegasus City? Show your work, and express your answer in a percentage (rounded to two decimal places).

probability ( $p$ ) such that the expected  
profit is the same in both locations

%

4. **Cities in Equilibrium—or Not** (7 points) Consider the following utility-per-worker curves for the cities Altair, Botein, and Canopus:



a. Starting with Altair at 20,000 workers, Botein at 20,000 workers, and Canopus at 20,000 workers what will the populations of these three cities be in their eventual stable equilibrium? Explain.

[Hint: There **is** enough information provided in the graph to answer this question.]

equilibrium population of Altair

equilibrium population of Botein

equilibrium population of Canopus

b. Explain why it is **not** an equilibrium if Altair has 60,000 workers, Botein has 100,000 workers, and Canopus has 200,000 workers.

c. Starting with Altair at 60,000 workers, Botein at 100,000 workers, and Canopus at 200,000 workers what will the populations of these three cities be in their eventual stable equilibrium? Explain.

[Hint: There **is** enough information provided in the graph to answer this question.]

equilibrium population of Altair

equilibrium population of Botein

equilibrium population of Canopus

d. Use the same graph of utility-per-worker curves but now consider a different situation entirely: Altair and Canopus each have 80,000 workers and the city of Botein doesn't exist at all. Is this situation a stable equilibrium or an unstable equilibrium? Explain.

[Hint: If Altair were to grow by 1000 workers while Canopus were to shrink by 1000 workers would there be self-reinforcing effects or self-correcting effects?]

5. **Rank (But Not the Album by The Smiths)** (2 points) Assume that the rank-size rule for cities is exactly true and that the second-largest city in a region has 3 million people.

a. How many people live in the region's largest city? Show your work.

population of largest city

b. How many people live in the region's third-largest city? Show your work.

population of third-largest city

c. How many people live in the region's twentieth-largest city? Show your work.

population of twentieth-largest city